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Fano resonances in a three-terminal nanodevice

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Abstract
Electron transport through a quantum sphere with three one-dimensional wires attached to it is investigated. An explicit form for the transmission coefficient as a function of the electron energy is found from first principles. The asymmetric Fano resonances are detected in the transmission of the system. The collapse of the resonances is shown to appear under certain conditions. A two-terminal nanodevice with an additional gate lead is studied using the developed approach. Additional resonances and minima of transmission are indicated in the device.

1. Introduction

Electron transport in nanoscale multiterminal ballistic devices has attracted considerable attention over the last decade. Rapid advances in nanoelectronic fabrication techniques have made possible the realization of electron waveguide devices with dimensions smaller than the elastic and inelastic scattering lengths of conduction electrons. Various interesting multiterminal nanoelectronic devices, such as the single electron transistor [1–3] and the three-terminal ballistic junction or Y-branch switch [4–6] have been proposed as promising alternatives for future low-power, high-speed switching devices. Recent theoretical studies reported transistor-like behaviour of various three-terminal molecule-based devices [7].

A number of theoretical and experimental works has been focused on the investigation of electron transport in multiterminal quantum systems. Three-terminal ballistic junctions were studied in [5, 6]. Electron transport in a three-terminal molecular wire connected to metallic leads was investigated in [8].

One of the interesting phenomena detected in these systems is Fano resonances in the transmission probability. Being a characteristic manifestation of wave phenomena in a scattering experiment, resonances have received considerable attention in recent electron transport investigations. A number of papers [9–13] have been devoted to the study of Fano resonances in transport through various quantum dots. Resonant tunnelling through quasi-one-dimensional channels with impurities is investigated in [14–17]. The temperature dependence of the zero-bias conductance of the single-electron transistor is considered in [3]. Coherent

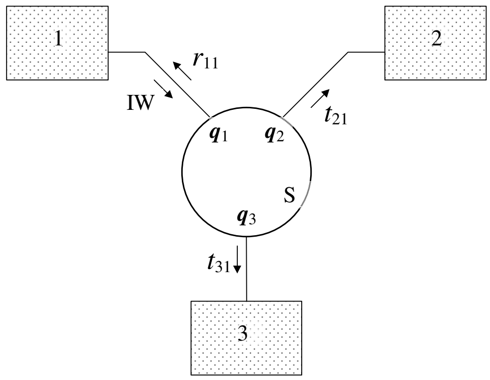

Figure 1. Scheme of the device. An incident wave (IW) originating from reservoir 1 is reflected back with amplitude r_{11} and scattered to reservoirs 2 and 3 with amplitudes t_{21} and t_{31}, respectively.

transport through a quantum dot embedded in an Aharonov–Bohm ring is investigated in [18]. The line shape of resonances in the overlapping regime is studied in [19].

Interference phenomena closely related to the Fano resonances have attracted considerable attention in the past few years. Those resonances are of universal nature and have been observed in various systems. We mention, for example, atom photoionization, electron and ion scattering, Raman scattering and so on. Recently, the line shape of resonances has been discussed in experiments on electron transport through mesoscopic systems [2, 3, 20]. It is shown in [21] that the same resonances occur in electron transport through a quantum nanosphere with two wires attached to it.

Recent progress in nanotechnology has made it possible to fabricate conductive two-dimensional nanostructures with spherical symmetry such as fullerenes and metallic spherical nanoshells. A number of works have been devoted to the theoretical study of electron transport on spherical surfaces [22–24]. The purpose of the present paper is an investigation of the electron transport through a three-terminal nanodevice consisting of a conductive nanosphere S with three one-dimensional wires attached to it at the points $\boldsymbol{q}_j$ ($j = 1, \ldots, 3$). We denote by $\boldsymbol{q}_j$ a set of spherical coordinates (θ_j, φ_j) of the point.

2. Hamiltonian and transmission coefficient

In our model, the wires are taken to be one-dimensional and represented by semiaxes $\mathbf{R}_j^+ = \{x : x \geqslant 0\}$ ($j = 1, \ldots, 3$). They are connected to the sphere by gluing the point $x = 0$ from $\mathbf{R}_j^+$ to the point $\boldsymbol{q}_j$ from S. We suppose $\boldsymbol{q}_i \neq \boldsymbol{q}_j$ for $i \neq j$. The scheme of the device is shown in figure 1. Here $t_{21}(E)$ and $t_{31}(E)$ are the transmission amplitudes of the electron wave and $r_{11}(E)$ is the reflection amplitude.

The Hamiltonian of a free electron in the wire is $H_j = p_x^2/2m^*$, where m^* is the electron effective mass and p_x is the momentum operator for the electron in wires. Electron motion on the sphere is described by the Hamiltonian $H_{\mathrm{S}} = \boldsymbol{L}^2/2m^*r^2$ where r is the radius of the nanosphere and $\boldsymbol{L}$ is the angular momentum operator. A wavefunction ψ of the electron in the device consists of four parts: ψ_{S}, ψ_1, ψ_2 and ψ_3, where ψ_{S} is a function on S and ψ_j ($j = 1, \ldots, 3$) are functions on $\mathbf{R}_j^+$. We note that in the general case ψ_{S} is not the eigenfunction of the operator H_{S}.

The Hamiltonian H of the whole system is a point perturbation of the operator

$$H_0 = H_{\mathrm{S}} \oplus H_1 \oplus H_2 \oplus H_3. \tag{1}$$

To define this perturbation we use boundary conditions at points of gluing. The role of the boundary values for the wavefunction $\psi_j(x)$ is played, as usual, by $\psi_j(0)$ and $\psi'_j(0)$. The zero-range potential theory shows that to obtain a non-trivial Hamiltonian on the whole system we must consider functions $\psi_{\rm S}(\boldsymbol{x})$ with a logarithmic singularity at points of gluing $\boldsymbol{q}_j$ [25]

$$\psi_{\rm S}(\boldsymbol{x}) = -u_j \frac{m^*}{\pi\hbar^2} \ln \rho(\boldsymbol{x}, \boldsymbol{q}_j) + v_j + {\rm o}(1) \tag{2}$$

as $\boldsymbol{x} \to \boldsymbol{q}_j$. Here u_j and v_j are complex coefficients and $\rho(\boldsymbol{x}, \boldsymbol{q})$ is the geodesic distance on the sphere between the points $\boldsymbol{x}$ and $\boldsymbol{q}_j$. It is known that the most general self-adjoint boundary conditions are defined by some linear relations between $\psi_j(0)$, $\psi'_j(0)$ and the coefficients u_j, v_j. Following [21] we will write this conditions in the form

$$\begin{aligned} v_j &= \sum_{k=1}^{3} \left[B_{jk} u_k - (\hbar^2/2m^*) A_{jk} \psi'_k(0) \right], \\ \psi_j(0) &= \sum_{k=1}^{3} \left[A^*_{kj} u_k - (\hbar^2/2m^*) C_{jk} \psi'_k(0) \right], \qquad j = 1, \ldots, 3. \end{aligned} \tag{3}$$

Here complex parameters A_{jk}, B_{jk} and C_{jk} form 3×3 matrices. The matrices B and C have to be Hermitian because the Hamiltonian H is a self-adjoint operator [25]. To avoid a non-local tunnelling coupling [24] between different contact points we will restrict ourselves to the case of diagonal matrices A_{jk}, B_{jk} and C_{jk} only.

According to the zero-range potential theory diagonal elements of the matrix B determine the strength of point perturbations of the Hamiltonian $H_{\rm S}$ at the points $\boldsymbol{q}_j$ on S [24]. These elements may be expressed in terms of scattering lengths λ_j^B on the corresponding point perturbations: $B_{jj} = m^* \ln(\lambda_j^B)/\pi\hbar^2$. Similarly, elements C_{jj} describe the strength of point perturbations at the points $x = 0$ in the wires and may be expressed in terms of scattering lengths λ_j^C by the relation $C_{jj} = -m^* \lambda_j^C / 2\hbar^2$ [21]. For convenience, we represent parameters A_{jj} in the form $A_{jj} = m^* \sqrt{\lambda_j^A} {\rm e}^{{\rm i}\phi_j}/\hbar^2$, where λ_j^A has the dimensions of length and ϕ_j is the argument of the complex number A_{jj}. Note that the effect of the scattering lengths λ_j^A, λ_j^B and λ_j^C on the electron transport has been discussed in [21]. In the present paper we concentrate our attention on the facts independent of the contact parameters.

To obtain transmission and reflection coefficients of the system one needs a solution of the Schrödinger equation for the Hamiltonian H. The function $\psi_1(x)$ in this solution is a superposition of incident and reflected waves while the functions $\psi_2(x)$ and $\psi_3(x)$ represent scattered waves. The wavefunction $\psi_{\rm S}(\boldsymbol{x})$ may be expressed in terms of the Green function $G(\boldsymbol{x}, \boldsymbol{y}; E)$ of the Hamiltonian $H_{\rm S}$ [21]

$$\begin{aligned} \psi_{\rm S}(\boldsymbol{x}) &= \sum_{j=1}^{3} \xi_j(E) G(\boldsymbol{x}, \boldsymbol{q}_j; E), \\ \psi_1(x) &= {\rm e}^{-{\rm i}kx} + r_{11}(E){\rm e}^{{\rm i}kx}, \\ \psi_2(x) &= t_{21}(E){\rm e}^{{\rm i}kx}, \\ \psi_3(x) &= t_{31}(E){\rm e}^{{\rm i}kx}. \end{aligned} \tag{4}$$

Here $k = \sqrt{2m^*E/\hbar^2}$ is the electron wavevector in wires and $\xi_j(E)$ are complex factors.

It is well known [26] that the Green function $G(\boldsymbol{x}, \boldsymbol{y}; E)$ may be expressed in the form

$$G(\boldsymbol{x}, \boldsymbol{y}; E) = -\frac{m^*}{2\hbar^2} \frac{1}{\cos(\pi t)} \mathcal{P}_{t-\frac{1}{2}}\left(-\cos\left(\rho(\boldsymbol{x}, \boldsymbol{y})/r\right)\right) \tag{5}$$

where $\mathcal{P}_\nu(x)$ is the Legendre function and $t(k) = \sqrt{r^2k^2 + 1/4}$.

Considering the asymptotics (2) of $\psi_S(\boldsymbol{x})$ from (4) near the point $\boldsymbol{q}_j$, we have

$$u_j = \xi_j(E), \qquad v_j = \sum_{i=1}^{3} Q_{ij}(E)\xi_i(E).$$

Here $Q_{ij}(E)$ is the so-called Krein $\mathcal{Q}$-function, that is a 3×3 matrix with elements

$$Q_{ij}(E) = \begin{cases} G(\boldsymbol{q}_i, \boldsymbol{q}_j; E), & i \neq j; \\ \lim\limits_{\boldsymbol{x}\to \boldsymbol{q}_j} \left[G(\boldsymbol{q}_j, \boldsymbol{x}; E) + \dfrac{m^*}{\pi\hbar^2} \ln \rho(\boldsymbol{q}_j, \boldsymbol{x}) \right], & i = j. \end{cases} \tag{6}$$

Using the asymptotic expression for the Legendre function in the vicinity of the point $x = -1$, we get the following form for diagonal elements of $\mathcal{Q}$-matrix [24]

$$Q_{jj}(E) = -\frac{m^*}{\pi\hbar^2} \left[\Psi\left(t(k) + \frac{1}{2} \right) - \frac{\pi}{2} \tan(\pi t(k)) - \ln(2r) + C_E \right], \qquad j = 1, \ldots, 3 \tag{7}$$

where $\Psi(x)$ is the logarithmic derivative of the Γ-function and C_E is the Euler constant.

Substituting (4) into (3), we get a system of six linear equations for ξ_j, r_{11}, t_{21} and t_{31}. For convenience, we introduce dimensionless elements of $\mathcal{Q}$-matrix

$$\tilde{Q}_{ij}(E) = (\hbar^2/m^*)(\mathcal{Q}_{ij}(E) - B_{ij}).$$

Solving the system of equations, we obtain

$$r_{11} = \frac{(k\lambda_1^C - 4\mathrm{i})\Delta_1}{(k\lambda_1^C + 4\mathrm{i})\Delta} \tag{8}$$

where

$$\Delta = \begin{vmatrix} \tilde{Q}_{11} - \frac{2k\lambda_1^A}{k\lambda_1^C + 4\mathrm{i}} & \tilde{Q}_{12} & \tilde{Q}_{13} \\ \tilde{Q}_{21} & \tilde{Q}_{22} - \frac{2k\lambda_2^A}{k\lambda_2^C + 4\mathrm{i}} & \tilde{Q}_{23} \\ \tilde{Q}_{31} & \tilde{Q}_{32} & \tilde{Q}_{33} - \frac{2k\lambda_3^A}{k\lambda_3^C + 4\mathrm{i}} \end{vmatrix}, \tag{9}$$

and

$$\Delta_1 = \begin{vmatrix} \tilde{Q}_{11} - \frac{2k\lambda_1^A}{k\lambda_1^C - 4\mathrm{i}} & \tilde{Q}_{12} & \tilde{Q}_{13} \\ \tilde{Q}_{21} & \tilde{Q}_{22} - \frac{2k\lambda_2^A}{k\lambda_2^C + 4\mathrm{i}} & \tilde{Q}_{23} \\ \tilde{Q}_{31} & \tilde{Q}_{32} & \tilde{Q}_{33} - \frac{2k\lambda_3^A}{k\lambda_3^C + 4\mathrm{i}} \end{vmatrix}. \tag{10}$$

The transmission amplitude t_{21} is given by

$$t_{21} = \frac{16k\sqrt{\lambda_1^A\lambda_2^A}\mathrm{e}^{\mathrm{i}(\phi_1-\phi_2)}[2k\lambda_3^A\tilde{Q}_{21} - (k\lambda_3^C + 4\mathrm{i})(\tilde{Q}_{21}\tilde{Q}_{33} - \tilde{Q}_{23}\tilde{Q}_{31})]}{\mathrm{i}(k\lambda_1^C + 4\mathrm{i})(k\lambda_2^C + 4\mathrm{i})(k\lambda_3^C + 4\mathrm{i})\Delta}. \tag{11}$$

Similarly, we can write

$$t_{31} = \frac{16k\sqrt{\lambda_1^A\lambda_3^A}\mathrm{e}^{\mathrm{i}(\phi_1-\phi_3)}[2k\lambda_2^A\tilde{Q}_{31} - (k\lambda_2^C + 4\mathrm{i})(\tilde{Q}_{31}\tilde{Q}_{22} - \tilde{Q}_{32}\tilde{Q}_{21})]}{\mathrm{i}(k\lambda_1^C + 4\mathrm{i})(k\lambda_2^C + 4\mathrm{i})(k\lambda_3^C + 4\mathrm{i})\Delta}. \tag{12}$$

We emphasize that the relation

$$|r_{11}|^2 + |t_{21}|^2 + |t_{31}|^2 = 1 \tag{13}$$

is valid for arbitrary energy E in compliance with the current conservation law.

The transmission coefficient $T_{21} \equiv |t_{21}|^2$ as a function of the dimensionless parameter kr is shown in figure 2. The figure corresponds to the case when contacts are placed equidistant on the great circle of the sphere. Denoting by ρ_{ij} the distance $\rho(\boldsymbol{q}_i, \boldsymbol{q}_j)$ between the points $\boldsymbol{q}_i$ and $\boldsymbol{q}_j$, we can represent the position of contacts by relation $\rho_{12} = \rho_{13} = \rho_{23} = 2\pi r/3$. In this case, the relation $t_{21} = t_{31}$ is valid for arbitrary energy due to the symmetry of system. Therefore T_{21} does not exceed the value $\frac{1}{2}$. All figures correspond to the case $\lambda_j^A = \lambda_j^B = \lambda_j^C = 0.1r$ for all j.

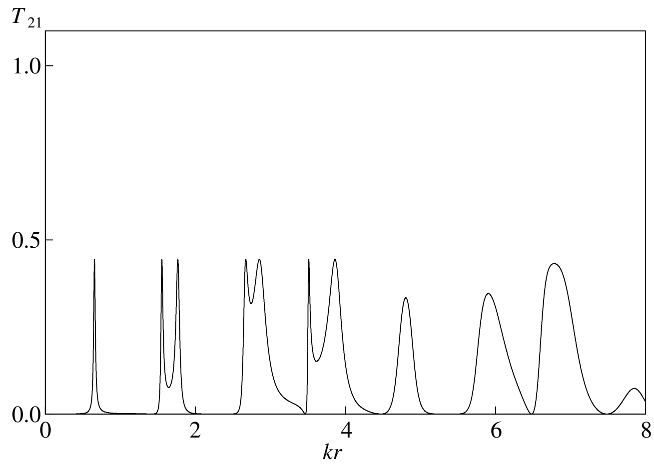

Figure 2. Transmission coefficient T_{21} as a function of the dimensionless parameter kr at $\rho_{12} = \rho_{13} = \rho_{23} = \frac{2}{3}\pi r$.

3. Fano resonances

It is evident from equation (11) that the transmission amplitude $t_{21}(E)$ has zeros of two different types. The zeros of the first type are stipulated by the poles of $Q_{ij}(E)$ and coincide with the eigenvalues E_l of the operator H_S. The denominator in (11) has a pole of the third order at $E = E_l$ while the numerator has a pole of the second order only. Hence, the transmission coefficient vanishes in these points.

The zeros of the second type are determined by the following equation:

$$2k\lambda_3^A \tilde{Q}_{21} - (k\lambda_3^C + 4\mathrm{i})(\tilde{Q}_{21}\tilde{Q}_{33} - \tilde{Q}_{23}\tilde{Q}_{31}) = 0. \tag{14}$$

The positions of the second-type zeros depend on the arrangement of $\boldsymbol{q}_j$ on the sphere in contrast to the positions of the first-type zeros.

We will show below that in the vicinity of the first-type zeros E_l the transmission coefficient has the form of the asymmetric Fano resonance. Consider the form of $Q_{ij}(E)$ near the point E_l

$$Q_{ij}(E) \simeq \frac{\alpha_{ij}}{E_l - E} + \beta_{ij}. \tag{15}$$

The residues α_{ij} of $Q_{ij}(E)$ at the point E_l may be expressed in terms of eigenfunctions of the operator H_S

$$\alpha_{ij} = \sum_{m=-l}^{l} Y_{lm}(\boldsymbol{q}_i) Y_{lm}^*(\boldsymbol{q}_j) \tag{16}$$

where $Y_{lm}(\boldsymbol{x})$ are the spherical harmonics.

Denote by $\tilde{\beta}_{ij}$ the modified matrix β

$$\tilde{\beta}_{ij} = \beta_{ij} - B_{ij} - \frac{2m^* k\lambda_j^A}{\hbar^2(k\lambda_j^C + 4\mathrm{i})}\delta_{ij}.$$

Substituting (15) into (11) and considering linear in $E - E_l$ approximation for the numerator and the denominator of (11), we obtain

$$t_{21}(E) \simeq \eta \frac{E - E_l}{E - E_R - \mathrm{i}\Gamma}. \tag{17}$$

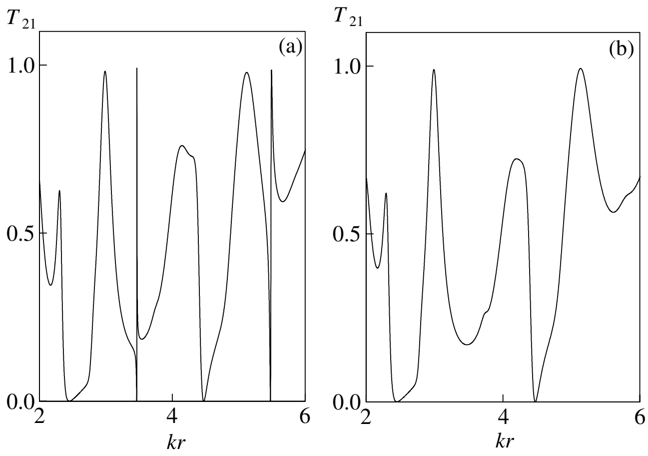

Figure 3. Transmission coefficient T_{21} as a function of the dimensionless parameter kr. (a) $\rho_{12} = 0.98\pi r$, $\rho_{13} = 0.52\pi r$, $\rho_{23} = 0.5\pi r$; (b) $\rho_{12} = \pi r$, $\rho_{13} = \rho_{23} = 0.5\pi r$ (collapse of the Fano resonances).

Here E_{R} determines the position of the asymmetric peak, Γ is the half-width of the resonance, and η is a normalization factor. It is evident from (17) that the transmission coefficient has the form of the Fano resonance near E_l. The parameters E_{R} and Γ of the Fano resonance are determined by

$$E_{\mathrm{R}} + \mathrm{i}\Gamma = E_l + \frac{\det\alpha}{\sum_{i,j}[\alpha_{ij}]^c\tilde{\beta}_{ij}} \tag{18}$$

where $[\alpha_{ij}]^c$ is the algebraic complement of α_{ij} in the matrix α. The normalization factor η is given by

$$\eta = \frac{16m^*k\sqrt{\lambda_1^A\lambda_2^A}\exp(\mathrm{i}(\phi_1 - \phi_2))}{\mathrm{i}\hbar^2(k\lambda_1^C + 4\mathrm{i})(k\lambda_2^C + 4\mathrm{i})\sum_{i,j}[\alpha_{ij}]^c\tilde{\beta}_{ij}}(\alpha_{23}\alpha_{31} - \alpha_{21}\alpha_{33}). \tag{19}$$

Note that the asymptotics (17) for $t_{21}(E)$ is valid for arbitrary parameters of contacts λ_j^A, λ_j^B and λ_j^C.

If $\det\alpha = 0$ at a given l then a collapse of the Fano resonance occurs near E_l. In this case, the pole and the zero of the transmission amplitude coincide and cancel each other (figure 3). Note that the second-type zeros remain on the plot of $T_{21}(E)$ in contrast to the situation considered in [21, 24].

To define the condition of the collapse we introduce three complex vectors $\boldsymbol{V}_j$ by the following equation

$$\boldsymbol{V}_j = \begin{pmatrix} Y_{l,l}(\boldsymbol{q}_j) \\ Y_{l,l-1}(\boldsymbol{q}_j) \\ \cdots \\ Y_{l,-l}(\boldsymbol{q}_j) \end{pmatrix}.$$

Matrix α is the Gram matrix for vectors $\boldsymbol{V}_j$ because $\alpha_{ij} = \langle\boldsymbol{V}_i|\boldsymbol{V}_j\rangle$. Hence, the condition $\det\alpha = 0$ is satisfied if and only if vectors $\boldsymbol{V}_j$ are linearly dependent.

If we choose a coordinate system on the sphere so that the points $\boldsymbol{q}_j$ were on the circle $\theta = \theta_0 =$ constant and fix the origin of the azimuthal angle φ at the point $\boldsymbol{q}_1$, then points $\boldsymbol{q}_j$

have the following coordinates

$$\boldsymbol{q}_1 = (\theta_0, 0), \qquad \boldsymbol{q}_2 = (\theta_0, \varphi_2), \qquad \boldsymbol{q}_3 = (\theta_0, \varphi_3).$$

Vectors $\boldsymbol{V}_j$ can be represented in the form

$$\boldsymbol{V}_1 = \begin{pmatrix} f_l^l \\ f_l^{l-1} \\ \ldots \\ f_l^{-l} \end{pmatrix}, \qquad \boldsymbol{V}_2 = \begin{pmatrix} f_l^l \mathrm{e}^{\mathrm{i}l\varphi_2} \\ f_l^{l-1} \mathrm{e}^{\mathrm{i}(l-1)\varphi_2} \\ \ldots \\ f_l^{-l} \mathrm{e}^{-\mathrm{i}l\varphi_2} \end{pmatrix}, \qquad \boldsymbol{V}_3 = \begin{pmatrix} f_l^l \mathrm{e}^{\mathrm{i}l\varphi_3} \\ f_l^{l-1} \mathrm{e}^{\mathrm{i}(l-1)\varphi_3} \\ \ldots \\ f_l^{-l} \mathrm{e}^{-\mathrm{i}l\varphi_3} \end{pmatrix},$$

where $f_l^m = C_{ml} P_l^{|m|}(\cos\theta)$, $P_l^{|m|}(x)$ are the Legendre polynomials, and C_{ml} are the normalization factors of the spherical harmonics.

Denote by M the $3 \times (2l+1)$ matrix composed of three vectors $\boldsymbol{V}_j$. The condition $\det\alpha \neq 0$ holds if and only if the rank of the matrix M is 3. In general, if points $\boldsymbol{q}_j$ are placed on the sphere in a random manner, all vectors $\boldsymbol{V}_j$ are linearly independent. If $\varphi_2 = \pi$ then all elements of M with different parity of m and l are equal to zero since $\theta_0 = \pi/2$ and $P_l^{|m|}(0) = 0$ for odd $m + l$. Elements of $\boldsymbol{V}_2$ with even $l + m$ in this case are equal to $(-1)^l f_l^m$. Hence the condition $\boldsymbol{V}_2 = (-1)^l \boldsymbol{V}_1$ is satisfied that directly implies $\det\alpha = 0$. Thus the collapse of the Fano resonances takes place if the points $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are antipodal on the sphere. This condition is independent of the position of $\boldsymbol{q}_3$.

It is evident that condition $\det\alpha = 0$ holds if any pair of three points $\boldsymbol{q}_j$ consists of antipodal points. But if $\varphi_3 = \pi$ or $|\varphi_2 - \varphi_3| = \pi$ then the normalization factor η vanishes because $\alpha_{23}\alpha_{31} - \alpha_{21}\alpha_{33} = 0$. In this case, the linear approximation for the denominator and the numerator of t_{21} is inapplicable, and the quadratic in $E - E_l$ terms in (11) must be taken into account. The equation similar to (17) may be obtained for t_{21} with

$$E_{\mathrm{R}} + \mathrm{i}\Gamma = E_l + \sum_{i,j} [\alpha_{ij}]^c \tilde{\beta}_{ij} \left(\sum_{i,j} \alpha_{ij} [\tilde{\beta}_{ij}]^c \right)^{-1}. \qquad (20)$$

In this case, the half-width Γ of the resonance is determined by the parameters $\tilde{\beta}_{ij}$, and the condition $\Gamma = 0$ requires a special choice of scattering lengths λ_j^A, λ_j^B and λ_j^C. Therefore, in general, the collapse of the Fano resonances appears when the points $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ only are antipodal on the sphere.

4. Two-terminal device with an additional gate lead

The dependence $T_{21}(E)$ is of particular interest because according to the Landauer–Büttiker formula the conductance of the system as a function of the chemical potential has the same form at zero temperature. For experimental observation of such a dependence one needs to change the electrochemical potential of electrons on the sphere relative to the Fermi energies in reservoirs. This may be realized by using an additional gate electrode near the sphere which is connected to the system through a potential barrier. Here we consider this additional gate lead as one-dimensional broken wire. The scheme of the studied device is shown in figure 4. In this case, one can shift energy levels of electrons on the sphere relative to the Fermi energy in the reservoirs 1 and 2 by changing the voltage V_{g}.

The electron wave outgoing from the sphere in this case reflects in the third wire and returns back completely. The solution of the Schrödinger equation for this system differs from (4) by the expression for $\psi_3(x)$

$$\psi_3(x) = t_{31}\mathrm{e}^{\mathrm{i}kx} + t_{31}\mathrm{e}^{\mathrm{i}\delta}\mathrm{e}^{-\mathrm{i}kx} \qquad (21)$$

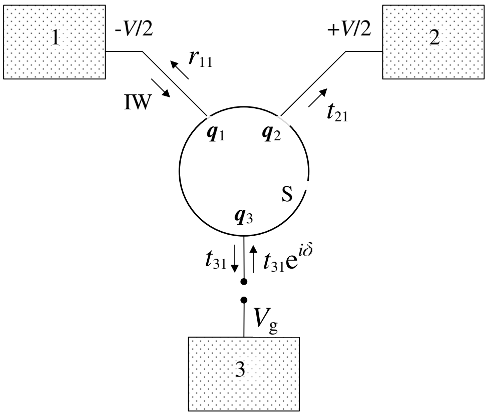

Figure 4. Scheme of the nanodevice with the break in the third wire. V is the bias voltage between reservoirs 1 and 2, and V_g is the gate voltage.

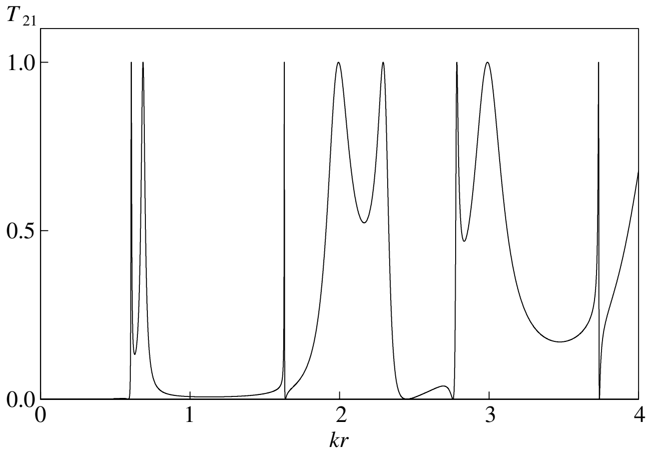

Figure 5. Transmission coefficient as a function of the dimensionless parameter kr in the case of the broken third wire at $\rho_{12} = \pi r$, $\rho_{13} = \rho_{23} = 0.5\pi r$ and $L = 0.57r$.

where $\delta = 2kL + \pi$ is the phase incursion and L is the distance between q_3 and the point of break.

The transmission coefficient in this case may be expressed in the form

$$t_{21} = \frac{16k\sqrt{\lambda_1^A\lambda_2^A}e^{i(\phi_1-\phi_2)}[2k\lambda_3^A\tilde{Q}_{21} - (k\lambda_3^C - 4\cot(\delta/2))(\tilde{Q}_{21}\tilde{Q}_{33} - \tilde{Q}_{23}\tilde{Q}_{31})]}{i(k\lambda_1^C + 4i)(k\lambda_2^C + 4i)(k\lambda_3^C - 4\cot(\delta/2))\tilde{\Delta}} \tag{22}$$

where

$$\tilde{\Delta} = \begin{vmatrix} \tilde{Q}_{11} - \frac{2k\lambda_1^A}{k\lambda_1^C + 4i} & \tilde{Q}_{12} & \tilde{Q}_{13} \\ \tilde{Q}_{21} & \tilde{Q}_{22} - \frac{2k\lambda_2^A}{k\lambda_2^C + 4i} & \tilde{Q}_{23} \\ \tilde{Q}_{31} & \tilde{Q}_{32} & \tilde{Q}_{33} - \frac{2k\lambda_3^A}{k\lambda_3^C - 4\cot(\delta/2)} \end{vmatrix}. \tag{23}$$

The dependence $T_{21}(E)$ for the case of the broken third wire is shown in figure 5. In contrast to the case considered above the height of the peaks can reach unity since there is no energy loss due to the outgoing of electrons into the third wire. Moreover the additional resonance peaks and minima arise because of the interference of electron waves in the broken wire.

5. Conclusions

Electron transport through a three-terminal nanodevice is considered. The transmission and reflection coefficients of the device are found by solving the Schrödinger equation. We have shown that, in the general case, the function $T_{21}(E)$ has zeros of two different types discussed in section 3. Zeros of the first type coincide with the eigenvalues E_l of the unperturbed electron Hamiltonian H_S on the sphere. The transmission coefficient $T_{21}(E)$ has the form of asymmetric Fano resonance in the vicinity of the first-type zeros. The parameters of the resonance E_R and Γ are determined by equation (18). If the points of contact $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are placed antipodal on the sphere then the collapse of the Fano resonances occurs. In this case, the first-type zeros disappear while the second-type zeros remain on the plot of $T_{21}(E)$ in contrast to the situation discussed in [21].

Using the developed approach we consider the two-terminal nanodevice with the additional gate electrode. Additional resonances and minima of transmission arise because of the interference of electron waves in the third wire.

Acknowledgment

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